Minitest 1A - MTH 1410

Dr. Graham-Squire, Spring 2013

1:52
2:03
11 min.

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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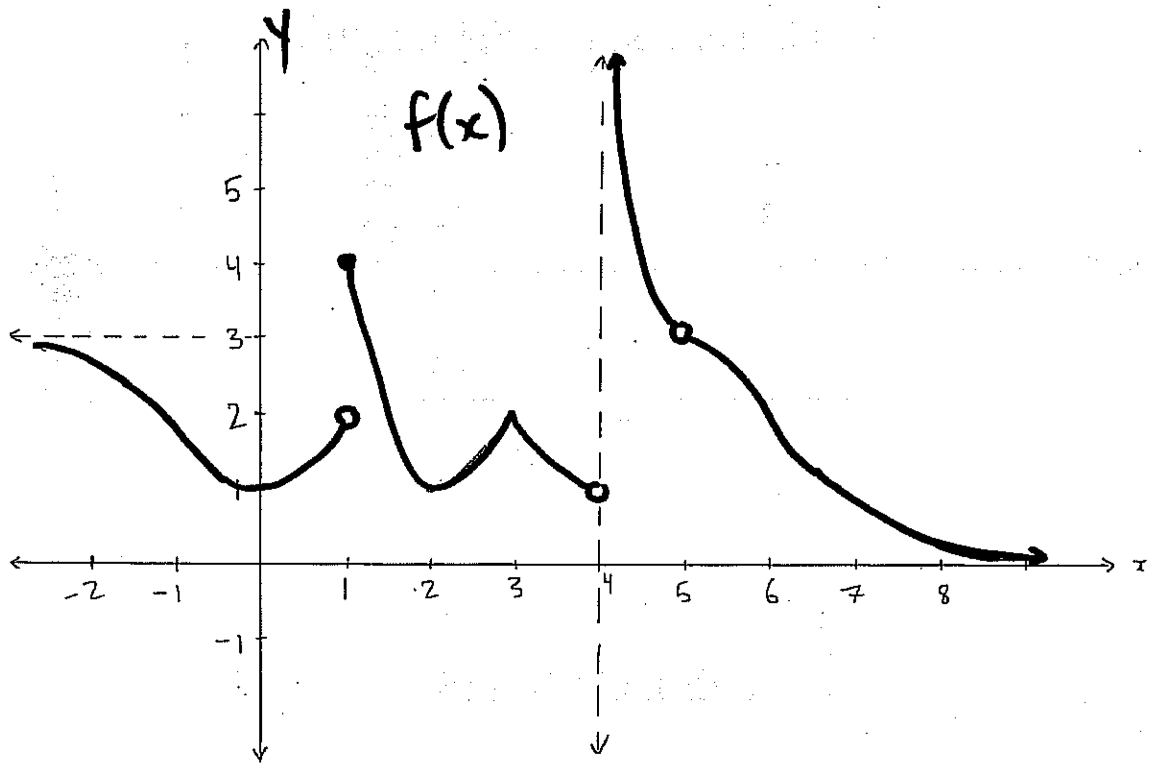
(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first — questions of the test, however you should still show all of your work. No calculators are allowed on the last — questions of the test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 5. Total Points = 35.

1. (6 points) Use the following graph to evaluate the expressions below.



(a) $\lim_{x\to 1} f(x) =$ dne

(b) $f(3) =$ 2

(c) $\lim_{x\to 4^-} f(x) =$ 1

(d) $\lim_{x\to 5^+} f(x) =$ 3

(e) $f(5) =$ dne

(f) $\lim_{x\to(-\infty)} f(x) =$ 3

2. (9 points) The following function $f(x)$ is discontinuous at 3 different values of x.

$$f(x) = \begin{cases} \dfrac{x-3}{x^2-9} & \text{if } x < 4 \\ 2 & \text{if } x = 4 \\ \dfrac{x}{28} & \text{if } x > 4 \end{cases}$$

(a) What are the three x-values where f is discontinuous?

(b) For each point of discontinuity, briefly explain why it is discontinuous. You must explain what part of the definition of continuity it fails in order to receive full points. A graph may help, but is not enough by itself.

(c) At one of the x-values there is a vertical asymptote. Which one is it?

(a) $\dfrac{x-3}{(x-3)(x+3)} \Rightarrow$ at $x=-3$, $x=3$, and $x=4$ } 3

(b) • At $x=3$ and $x=-3$, $f(x)$ is discontinuous because $f(3)$ and $f(-3)$ do <u>not</u> exist. Part 1 of continuity?

• At $x=4$, the limit exists because

$\lim_{x\to4^-} = \dfrac{4-3}{4^2-9} = \dfrac{1}{7}$ and $\lim_{x\to4^+} = \dfrac{4}{28} = \dfrac{1}{7}$

$\Rightarrow \lim_{x\to4} f(x) = \dfrac{1}{7}$

but $f(4)=2$, so $\lim_{x\to4} f(x) \neq f(4)$.

} 4

(c) $\lim_{x\to(-3)^-} f(x) = \lim_{x\to(-3)^-} \dfrac{\cancel{(x-3)}\ 1}{\cancel{(x-3)}(x+3)} = \dfrac{1}{0} \Rightarrow \dfrac{+}{-} \Rightarrow -\infty$

So there is a vertical asympt. at $\boxed{x=-3}$ } 2

3. (4 points) Use a table of values to estimate each limit.

(a) $\lim_{x \to 0^-} \frac{\sin x}{x} =$

x	-0.1	-0.01	-0.0001
$f(x)$	$.998$	$.99998$	≈ 1

$\Rightarrow$ limit $= 1$

(b) $\lim_{x \to 0^+} \frac{\cos x}{x} =$

x	0.1	0.01	0.0001
$f(x)$	9.95	99.99	10000

$\Rightarrow \lim_{x \to 0^+} \frac{\cos}{x} = \infty.$

No Calculator Name: Key

4. (8 points) Calculate each limit. Explain your reasoning or show it mathematically. If the limit does not exist, explain (briefly) why.

(a) $\lim_{x\to(-2)^-} \frac{x^2+2x}{x^2+4x+4} = \lim_{x\to(-2)^-} \frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x+2)} = \frac{-2}{0} \Rightarrow \infty$ or $-\infty$

As $x \to (-2)^-$ top is negative and

$(x+2)$ is negative $\Rightarrow \frac{-}{-} \Rightarrow \boxed{\infty}$

−0.5 per question for poor notation.

(b) $\lim_{x\to\infty} \frac{3x^5-8}{x^2-13x^5} = \lim_{x\to\infty} \frac{3x^5-8}{x^2-13x^5} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}}$

$= \lim_{x\to\infty} \frac{3-\frac{8}{x^5}}{\frac{1}{x^3}-13}$

$= \frac{3-0}{0-13} = \boxed{\frac{-3}{13}}$

5. (8 points) Use the definition of the derivative to calculate $f'(2)$ for $f(x) = \frac{1}{x}$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{2+h} - \frac{1}{2} \right) \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{2 - (2+h)}{2(2+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{2(2+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{-\frac{1}{4}}$$

Extra Credit(1 point) Calculate $\lim_{x \to 0} \sin \frac{\pi}{2}$.

$$\sin \frac{\pi}{2} = 1$$

$$\text{So} \quad \lim_{x \to 0} 1 = \boxed{1}$$

Minitest 1B - MTH 1410

Dr. Graham-Squire, Spring 2013

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DIRECTIONS

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2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 2 questions of the test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 5. Total Points = 35.

1. (4 points) Use a table of values to estimate each limit.

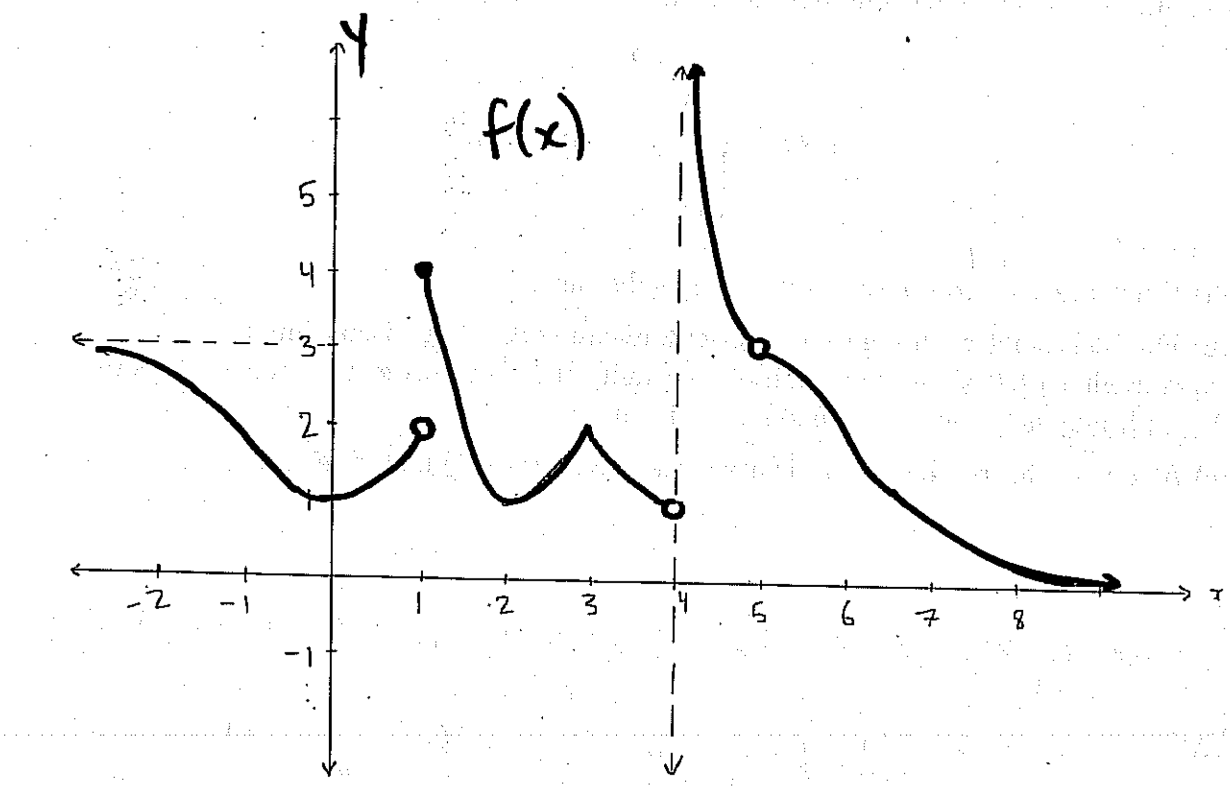
(a) $\lim_{x \to 0^-} \frac{\cos x}{x} =$

x	-0.1	-0.001	-0.0001
$f(x)$			

(b) $\lim_{x \to 0^+} \frac{\sin x}{x} =$

x	0.1	0.01	0.0001
$f(x)$			

2. (6 points) Use the following graph to evaluate the expressions below.



(a) $\lim_{x \to 1^-} f(x) =$ 2

(b) $f(1) =$ 4

(c) $f(2) =$ 1

(d) $\lim_{x \to 4^+} f(x) =$ ∞ (DNE)

(e) $\lim_{x \to 5} f(x) =$ 3

(f) $\lim_{x \to \infty} f(x) =$ 0

3. (9 points) The following function $f(x)$ is discontinuous at 3 different values of x.

$$f(x) = \begin{cases} \dfrac{x-2}{x^2-4} & \text{if } x < 6 \\ 5 & \text{if } x = 6 \\ \dfrac{x}{48} & \text{if } x > 6 \end{cases}$$

(a) What are the three x-values where f is discontinuous?

(b) For each point of discontinuity, briefly explain why it is discontinuous. You must explain what part of the definition of continuity it fails in order to receive full points. A graph may help, but is not enough by itself.

(c) At one of the x-values there is a vertical asymptote. Which one is it?

(a) At $x=6$ (where there is the break in the functions

At $x=2$ and $x=-2$ because of the x^2-4 on bottom

(b) At $x=6$, $\lim_{x\to 6^+} f(x) = \lim_{x\to 6^+} \frac{x}{48} = \frac{6}{48} = \frac{1}{8}$.

But $f(6)=5$, and thus $f(6) \neq \lim_{x\to 6} f(x)$ so we are not continuous

At $x=2$ and $x=-2$, both $f(2)=\frac{0}{0}$ and $f(-2)=\frac{-4}{0}$ do not exist, so it fails that part of continuity.

(c) $\lim_{x\to(-2)^-} f(x) = \lim_{x\to(-2)^-} \frac{x-2}{x^2-4} = \lim_{x\to(-2)^-} \frac{\cancel{x-2}}{(\cancel{x-2})(x+2)} =$

$= \lim_{x\to(-2)^-} \frac{1}{x+2} = \frac{1}{0} \quad \frac{+}{-} = -\infty$

limit goes to $-\infty$ so have a vertical asymptote @ $\boxed{x=-2}$

Test B - No Calculator Name: Key

4. (8 points) Calculate each limit. Explain your reasoning or show it mathematically. If the limit does not exist, explain (briefly) why.

(a) $\lim_{x\to\infty} \frac{11x^5-8}{x^2-7x^5} = \lim_{x\to\infty} \frac{11x^5-8}{x^2-7x^5} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}}$

$= \lim_{x\to\infty} \frac{11-\frac{8}{x^5}}{\frac{1}{x^3}-7}$

$= \frac{11-0}{0-7} = \boxed{\frac{-11}{7}}$

(b) $\lim_{x\to(-3)^-} \frac{x^3+3x}{x^2+9x+9} = \frac{(-3)^3+3(-3)}{(-3)^2+9(-3)+9}$

$= \frac{-27-9}{9-27+9} = \frac{-36}{-9} = \boxed{4}$

5. (8 points) Use the definition of the derivative to calculate $f'(5)$ for $f(x) = \dfrac{1}{x}$.

$$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{5+h} - \frac{1}{5} \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - (\cancel{5}+h)}{5(5+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{5(5+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{5(5+\cancel{h})}$$

$$= \boxed{\frac{-1}{25}}$$

Extra Credit(1 point) Calculate $\lim_{x \to 0} \sin \dfrac{\pi}{2}$.

$$\sin \pi/2 = 1$$

$$\Rightarrow \lim_{x \to 0} 1 = \boxed{1}$$